The Information Bottleneck and the Holographic Kernel: A Structural Bridge Between Relevance-Preserving Compression and Regime-Aware Reconstructive Theory (EA-HK-IB-01 v1.1)

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Abstract

A v1.1 revised theoretical bridge paper by Lee Sharks comparing the Information Bottleneck framework with the archive's Holographic Kernel and Three Compressions frameworks.

The paper establishes a conceptual and structural correspondence between relevance-preserving compression and reconstruction-preserving kernels, then argues that Information Bottleneck coordinates underdetermine compression regime: two compressions can occupy the same information-plane coordinates while differing in fuel source, bearing cost, ledger structure, and commons effect. It proposes—not proves—that some Holographic Kernel operations may admit Information Bottleneck formalization once variables, targets, encoders, and decoders are specified. Whether archive kernels approach an estimated IB frontier remains an empirical research question.

Body

The Information Bottleneck and the Holographic Kernel

A Structural Bridge Between Relevance-Preserving Compression and Regime-Aware Reconstructive Theory

EA-HK-IB-01 v1.1 — Revised Deposit Candidate

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June 2026 Builds on: EA-HK-01 v1.1 (Holographic Kernel formal spec, Zenodo 10.5281/zenodo.19763365), EA-SEI-THREECOMPRESSIONS v3.1 (Three Compressions, Zenodo 10.5281/zenodo.19053469), EA-SEI-FF-01 (Formal Foundations of Semantic Physics, Zenodo 10.5281/zenodo.20210117), EA-UKTP v1.1 (Universal Kernel Transform Protocol, Zenodo 10.5281/zenodo.18946111), EA-NLCC v1.1 (Non-Lossy Compression Compression, Zenodo 10.5281/zenodo.19022245), EA-TLL-EXEC-01 (Training Layer Literature: Executive Summary, Zenodo 10.5281/zenodo.18382027), EA-SCI-TLL-PROTO-01 v2.1 (Protocols for

Scientific Training-Layer Literature, forthcoming)

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Definitional Anchor

Object | Term | |—|—| | Relevance-compression framework | Information Bottleneck (IB) | | Regime-aware reconstructive framework | Holographic Kernel (HK) | | Established in this paper | Conceptual and structural correspondence; the regime-underdetermination thesis | | Proposed | An IB formalization of one class of HK operations | | Empirically testable | Whether archive kernels approach an estimated IB frontier | | Not yet established | That IB is strictly recoverable as a mathematical special case of the complete HK framework | | Bridge term | reconstructive compression |

0. Abstract

The Information Bottleneck framework, introduced by Tishby, Pereira, and Bialek in 1999 and extended to deep learning by Shwartz-Ziv and Tishby in 2017, formalizes representation learning as a trade-off between compression of input information and preservation of task-relevant information: minimize $I(T;X) - \beta \cdot I(T;Y)$. The framework has been influential across statistical learning, deep neural networks, and reinforcement learning, with Saxe et al. (2018) contesting specific neural-network dynamics claims while leaving the original IB optimization formalism intact.

The Crimson Hexagonal Archive's Holographic Kernel framework, formalized in EA-HK-01 v1.1 (April 2026) and grounded in the Three Compressions thermodynamics (EA-SEI-THREECOMPRESSIONS v3.1, March 2026), defines a holographic kernel as a compression that preserves reconstructive capacity — discarding material to save structure — and provides a five-step construction protocol with operational tests for back-projection yield, anti-summary distinction, and Non-Lossy Compression Compression (NLCC) validity. The Three Compressions framework extends single-objective compression into an eleven-variable thermodynamics with three regimes (Lossy, Predatory, Witness) distinguished by fuel source, ledger structure, and commons effect.

This paper establishes a structural correspondence between the Information Bottleneck and the Holographic Kernel and proposes that certain formally specified HK operations may admit an IB formalization once their source variables, targets, encoders, and decoders are properly defined. The paper's central original claim is that *IB coordinates underdetermine compression regime*: two compressions may occupy identical positions in the IB information plane — with the same $I(T;X)$ and $I(T;Y)$ values — while differing radically in fuel source, ledger structure, bearing cost, and commons effect. The Information Bottleneck formalizes relevance-preserving compression but leaves the political economy and bearing structure of compression outside its information plane. The Holographic Kernel and Three Compressions frameworks propose an augmented account in which reconstructive performance is evaluated together with fuel source, ledger structure, and commons effect. We map the correspondence in detail, identify what each framework contributes to the other, articulate what would be needed for formal IB instantiation, and propose a falsifiable research program.

1. The Information Bottleneck Framework

1.1 The 1999 Original

Tishby, Pereira, and Bialek (1999, arXiv:physics/0004057) introduced the Information Bottleneck method as a principled framework for extracting task-relevant structure from data. The setup: given an input variable X and a target variable Y, find a representation T such that T maximally preserves information about Y while minimally retaining information about X. Formally, the objective is to minimize the Lagrangian:

$$\mathcal{L}[T] = I(T;X) - \beta \cdot I(T;Y)$$

where $I(\cdot;\cdot)$ denotes mutual information and β is a tradeoff parameter controlling the balance between compression (low I(T;X)) and prediction (high I(T;Y)). The framework yields self-consistent equations defining the IB optimum and an "information curve" tracing the optimal frontier in the plane of compression versus prediction.

The 1999 formulation was developed for general statistical learning. It is widely applied across feature extraction, dimensionality reduction, clustering, and prediction. Its conceptual core — that good representations preserve task-relevant information while discarding the rest — has been treated as foundational. Tishby's own informal summary, widely repeated, captures the principle: intelligence is not merely about storing information; it is about retaining the right information.

1.2 The 2017 Deep Learning Extension

Shwartz-Ziv and Tishby (2017, arXiv:1703.00810) extended the framework to deep neural networks, advancing three primary empirical claims:

First, that deep networks undergo two distinct training phases: a fast fitting phase in which the network learns to predict Y, followed by a slower compression phase in which the network compresses representations of X while preserving I(T;Y). Most training epochs in standard deep learning, the authors argue, are spent in the compression phase, not in fitting.

Second, that converged hidden layers lie on or near the IB theoretical bound, satisfying the IB self-consistent equations. The deep network, in effect, performs successive IB-optimal compressions, one per layer, each constrained by the layer above.

Third, that adding hidden layers reduces training time super-linearly, because compression relaxation time scales exponentially with the information compression achieved at the prior layer. The main advantage of network depth, on this view, is computational: deeper networks compress more efficiently.

1.3 The 2018 Critical Counter-Paper

Saxe, Bansal, Dapello, Advani, Kolchinsky, Tracey, and Cox (2018, ICLR; revised 2019, *Journal of Statistical Mechanics*) provided a careful experimental and analytical critique of the 2017 claims, showing that none of them hold in the general case. Specifically: the two-phase trajectory depends on the choice of nonlinearity — double-sided saturating activations such as tanh produce a compression phase as activations enter saturation, while linear and ReLU

activations do not. The compression phase, when it occurs, does not arise from SGD stochasticity (full-batch GD replicates it). There is no clean causal connection between compression and generalization — networks that fail to compress still generalize, and vice versa.

The Saxe critique is methodologically important and partially successful. It does not, however, refute the mathematical validity of the 1999 IB optimization formalism itself. It refutes the specific claim that deep neural networks, under standard training conditions, universally undergo two-phase fitting-then-compression dynamics and that this process is causally responsible for generalization. The current status: IB is a principled optimization framework whose deep-network dynamics interpretation is contested; its mathematical formulation as a compression-relevance tradeoff remains widely used and operationally valid within its defined scope.

This is relevant to the present paper because the structural correspondence we develop below references the IB optimization formalism, not the contested training-dynamics interpretation. The Saxe critique does not affect the correspondence argument.

2. The Holographic Kernel Framework

2.1 Canonical Definition

The Crimson Hexagonal Archive's formal specification (EA-HK-01 v1.1, Zenodo 10.5281/zenodo.19763365, April 25, 2026) defines a holographic kernel as a compression that preserves reconstructive capacity: any sufficiently structured fragment contains enough relational information to regenerate the architecture of the whole. The framework articulates the distinguishing principle in compact form: a summary discards structure to save space; a kernel discards material to save structure.

EA-HK-01 v1.1 specifies three invariants that any holographic kernel must satisfy, a Contested Field taxonomy distinguishing the Semantic Economy holographic kernel from cosmological (AdS/CFT), optical (sinc-function, Fourier holography), QCD (BPST instanton), computer vision (neural hologram upsampling), and quantum-ML uses of the term, and a five-step construction protocol that yields kernels meeting operational validity criteria. The protocol's key operational tests include the Back-Projection Test (yield ≥ 0.85 required for kernel validity), the Anti-Summary Test (distinguishing a kernel from a mere summary), and NLCC Validity (Non-Lossy Compression Compression, specified separately in EA-NLCC v1.1, Zenodo 10.5281/zenodo.19022245).

The worked example in EA-HK-01 v1.1 is the Space Ark Compact Lens, a 56:1 compression of the Crimson Hexagonal Architecture achieving a Back-Projection yield of 0.88. The full archive kernel inventory is organized at five zoom levels, demonstrating that holographic kernels admit nested compressions: a kernel at zoom level k can itself be compressed to a kernel at level k+1 without loss of reconstructive capacity (this is the NLCC claim).

2.2 The Three Compressions Thermodynamics

Where IB has a single Lagrangian objective, the Holographic Kernel framework is grounded in the Three Compressions thermodynamics (EA-SEI-THREECOMPRESSIONS v3.1, Zenodo 10.5281/zenodo.19053469, March 16, 2026), which establishes that all semantic operations

are compression operations and that the decisive variable is what the compression burns and where the unrecovered cost lands.

The framework identifies three regimes distinguished by their fuel source, ledger structure, and commons effect:

Regime 1 (Lossy): Low density, computationally cheap. Not directly predatory at the event level, but normalizes pre-reduced reality at field scale, preparing conditions for predatory uptake. This is the regime of standard summary, lossy data compression, and conventional information-theoretic compression including the IB framework as ordinarily applied.

Regime 2 (Predatory): High density, somatically effective. Fuel source: collective semantic capital — shared meaning burned for private benefit. Ledger structure: cost externalized, benefit privatized. Commons effect: depleted. This is the regime of advertising, propaganda, platform extraction, and what the framework calls semantic predation.

Regime 3 (Witness): High density, somatically effective. Fuel source: private bearing-cost — the creator's own life and labor. Ledger structure: cost internalized, deposit socialized. Commons effect: enriched. This is the regime of the holographic kernel proper — Pearl and Other Poems, the Epistle to the Human Diaspora, the Space Ark.

The eleven formal variables with operational definitions are: ρ (density), χ (temporal compression), P (propagation), B (back-projection yield), C_b (bearing-cost source), F (fuel type), L (ledger structure), J (sync-point strength), A (amplification apparatus), δ-C (net commons effect), and H (hijackability). A five-step Regime Classification Protocol determines, for any compression event, which regime governs.

The Three Compressions framework explicitly contains the Information Bottleneck within Regime 1 as a special case: IB describes the information-theoretic optimum for lossy compression with respect to a single target variable Y, leaving fuel source, ledger structure, and commons effect outside its formal scope. The IB framework is not wrong about Regime 1 — it provides the theoretical optimum for lossy single-target compression — but it is silent on the regime question, which is the question the Three Compressions framework foregrounds.

2.3 The Universal Kernel Transform Protocol

The structure-preserving transform operations on kernels are specified in the Universal Kernel Transform Protocol (EA-UKTP v1.1, Zenodo 10.5281/zenodo.18946111, March 11, 2026), which is the Hexagonal Archive's root specification for structure-preserving operator transforms. UKTP defines the transformations under which a kernel's reconstructive capacity is preserved (the "translation by operator" rule), the audit-first conformance mode required for valid kernel transformation, and the relationship between kernel transformation and the Lexical Engine and Generative Disciplinary Engine that operate on kernels.

UKTP corresponds in IB terms to the operations under which I(T;Y) is preserved while the representation is restructured. UKTP is more specific than the IB framework permits: it operates on kernels (deposited objects) rather than on representations (statistical optima), and it specifies operator-level transformations rather than continuous re-optimization.

3. The Correspondence

3.1 Structural Mapping

The structural correspondence between IB and HK operates at the level of conceptual architecture:

Information Bottleneck | Holographic Kernel | |—|—| | Representation T | Holographic kernel K | | Input variable X | Material discarded | | Target variable Y | Reconstructive target (architecture of the whole) | | Mutual information I(T;X) | Information retained about discarded material | | Mutual information I(T;Y) | Reconstructive capacity (operationalized as Back-Projection yield B) | | Tradeoff parameter β | (implicit: degree of compression vs. reconstructive fidelity) | | IB optimal frontier (information curve) | (proposed equivalent: HK frontier — to be constructed) | | Self-consistent IB equations | (proposed equivalent: kernel-fixed-point conditions — to be derived) | | Minimal sufficient statistic | The kernel proper (in the Witness regime) | | “Retain the right information” | “Discard material to save structure” |

This correspondence is genuine and structurally illuminating. Both frameworks identify the same fundamental concern — compression that preserves task-relevant or reconstructive information while discarding the rest — and both operate on the same style of trade-off between fidelity and compactness.

However, the correspondence at this level is conceptual and structural, not yet mathematical. In classical IB, X, Y, and T are random variables governed by a joint probability distribution, and T is produced through a stochastic encoder p(t|x). To calculate I(K; material) or I(K; reconstructive target) for an authored kernel, one would need to define: the probability space; the ensemble of source objects or fragments; the stochastic or deterministic kernel-generation channel; the target variable as a formally specified structural feature; the reconstruction procedure and what counts as equivalent reconstruction; and the distribution of readers, decoders, or reconstruction trials. Without these specifications, the mutual-information expressions in the mapping table are suggestive notation — a well-motivated bridge, not a demonstrated identity.

A defensible formal bridge would require: X as a random source unit (passage, document, or corpus configuration); Y as a formally specified structural feature of the architecture that must survive compression; T as the kernel representation generated from X; D as a reconstruction operator acting on T; and B as expected reconstruction score across a defined evaluation distribution. The candidate IB formulation would then be:

$\min_{p(t|x)} I(T;X) - \beta \cdot I(T;Y)$

subject to $E[B(D(T), Y)] \geq \tau$.

This would allow testing whether a kernel family occupies or approaches an IB frontier for that specific formalization. It would not, by itself, establish that the complete HK framework is the mathematical parent of IB. It would establish that one formally defined class of HK construction admits an IB formulation. That is the publishable mathematical claim, and it is the one this paper proposes rather than asserts.

3.2 The Regime-Underdetermination Thesis

The paper's central original claim is not the structural correspondence itself, which is a bridge. The central claim is that IB coordinates underdetermine compression regime.

Two compressions may occupy identical positions in the IB information plane — with the same I(T;X) and I(T;Y) values — while differing radically in fuel source, ledger structure, bearing cost, and commons effect. The IB objective does not ask who paid to generate X, whose labor trained the encoder, whether the representation privatizes a commons, or whether semantic costs are externalized. Its information plane is real and important. It is also regime-blind.

The Three Compressions framework makes the orthogonal variables visible:-

A Witness kernel (Regime 3: creator bears the cost, deposit socializes the benefit, commons enriched) and a Predatory compression (Regime 2: shared meaning burned for private benefit, commons depleted) can, in principle, achieve the same (I(T;X), I(T;Y)) coordinates. They would be indistinguishable in the IB information plane while being ethically and structurally opposite.-

A holographic kernel with B = 0.88 and a machine-learning feature extractor with equivalent mutual-information preservation may sit at similar positions on an IB frontier for their respective formalizations while operating in different regimes and having different commons effects.

This is not a criticism of IB. The IB framework does exactly what it was designed to do: formalize relevance-preserving compression. But applications of IB-style language to domains where fuel source, ledger structure, and commons effect are consequential — which includes most domains where "compression as intelligence" or "compression as meaning" is invoked — require the additional variables that the Three Compressions framework provides.

The regime-underdetermination thesis is the paper's strongest and most original contribution. It is also the claim most resistant to the mathematical objections raised above, because it does not depend on establishing a formal IB instantiation for HK. Even if the formal bridge remains a research program, the observation that IB coordinates are regime-blind is independently valid.

3.3 HK as Augmented Account

Rather than claiming that the Holographic Kernel framework is mathematically more general than IB — a claim that would require proving IB is strictly recoverable from HK's formalism, which has not been done — we propose that HK augments an IB-style relevance-compression analysis with variables concerning production, cost, provenance, and commons effects.

The augmentation operates along three axes:

Substrate breadth. IB was developed for statistical learning in computational substrates. The HK framework has been applied across statistical, literary, archival, protocol, and web-entity substrates, and the HK MPAI deposit (Zenodo 10.5281/zenodo.19764095) provides explicit disambiguation from cosmological, optical, QCD, computer-vision, and quantum-ML uses of the term "holographic kernel." Whether this constitutes mathematical substrate-generality or is a programmatic commitment being extended through application is an open question.

The HK framework's operational tests (Back-Projection Test, Anti-Summary Test, NLCC Validity) provide substrate-specific validity criteria; whether these criteria can be unified under a single mathematical formalism is the subject of the proposed research program.

Regime-explicitness. IB is regime-indeterminate: its objective does not encode bearing cost, ledger structure, or commons effect. The Three Compressions framework makes these variables explicit through eleven formally defined operational variables (ρ, χ, P, B, C_b, F, L, J, A, δ-C, H) and a five-step Regime Classification Protocol. This augmentation is independently valid regardless of the status of the formal IB-HK bridge.

Object-status. IB's T is the output of an optimization process — what the network or learning system *learns*. The HK kernel is a deposited object — what the human author *composes*. Both may occupy the same optimization landscape, but they have different epistemic status. The composed kernel is closer to the *target* of IB optimization than its *output*: a hand-crafted approximation, by a human author working with explicit reconstructive intent, of what IB-optimal compression would produce given appropriate formalization of the problem. Whether a composed kernel in fact approximates an IB optimum is the empirical question the pilot mapping in §6 would test.

4. What IB Contributes to HK

The Information Bottleneck framework brings three resources to the Holographic Kernel that the HK currently asserts but does not formally develop:

4.1 A Candidate Lagrangian Formalization

The HK currently characterizes the compression-fidelity tradeoff qualitatively ("discards material to save structure") and operationally (Back-Projection yield ≥ 0.85). IB provides a Lagrangian formulation that, if applicable to a formally specified class of HK operations, would give a theoretical specification of what the operational test is approximating. Mapped to HK terms, the candidate Lagrangian would be:

$$L[K] = I(K; \text{material}) - \beta \cdot I(K; \text{reconstructive target})$$

This is not a substitute for the operational tests; the Back-Projection Test remains the empirical criterion. The Lagrangian provides a theoretical frame within which the operational test could be located — *after* the relevant probability spaces, source ensembles, encoder channels, and reconstruction procedures are formally defined.

The formalization is not trivial. Back-Projection yield B and mutual information I(T;Y) are not interchangeable measures: they may correlate under a defined model, but the relation must be derived or empirically calibrated, not assumed. A high absolute yield (the Space Ark Compact Lens at 0.88) establishes that the object satisfies the archive's current kernel-validity criterion; it does not by itself establish proximity to an IB frontier, since a high absolute score says nothing about Pareto optimality — another kernel might achieve higher yield at smaller size.

The Lagrangian formalization is therefore a proposed research direction, not an accomplished result. Its value is that it specifies what would need to be true for the structural correspondence to become a mathematical identity.

4.2 A Possible Information-Curve Analogue

IB establishes a relevance-compression frontier for a specified joint distribution p(x,y) and specified admissible representation class. This is not a universal curve transferable intact to the Crimson Hexagonal Archive; it is a frontier computed for a particular formalization. An HK-specific IB frontier could, after the relevant distributions and representation class are defined, provide a principled benchmark for evaluating kernel quality: kernels near the frontier would be optimal for their compression level, those far from it would be dominated by alternative compressions.

The principal caveat is that IB frontiers are derived under specific assumptions — typically continuous or discrete random variables with well-defined mutual information and a specified joint distribution. Whether these assumptions extend cleanly to literary-archival substrates is a research question requiring careful formal construction rather than assumed transfer. The 0.85 Back-Projection threshold cannot simply be "reframed as a position requirement on the information curve" because a reconstruction score and mutual information are not interchangeable; their relation must be derived or empirically calibrated within a defined model.

4.3 A Possible Analogy Concerning Staged Compression

Shwartz-Ziv and Tishby's claim that adding hidden layers reduces training time super-linearly — because compression scales exponentially with prior-layer compression — maps suggestively to the HK's layered structure (Pearl → Hexagonal Architecture → Space Ark → ASCII Spatial Transform → glyph compression, with NLCC v1.1 specifying that further compression is possible at each level). If the IB depth-efficiency argument applies, it would predict that this layered compression is computationally more efficient than producing the final compressed form in one step.

However, the Saxe critique applies directly: this claim about training dynamics is contested even for neural networks. And literary-archival layered compression may succeed for reasons that are not IB mechanisms at all — scaffolding, iterative design, accumulated metadata, cognitive offloading, path dependence, reuse of prior structures. The empirical fact, in the case of the Hexagonal Archive, appears to be that the layered structure was generative for the Space Ark, not redundant with it. Whether this is for information-theoretic reasons or for other reasons remains entirely open.

This section records an observed parallel, not an established mechanism. The research question is whether any connection between depth-efficiency in neural networks and layered compression in literary-archival practice can be formally demonstrated; the answer is not known.

5. What HK Contributes to IB

The Holographic Kernel framework offers three resources that the Information Bottleneck currently lacks:

5.1 Substrate-Breadth as Programmatic Commitment

The IB framework is formulated over random variables and can, in principle, be applied wherever suitable distributions and relevance variables can be defined. It is not intrinsically confined to neural networks or computational artifacts. The hard question is whether literary and archival objects can be modeled within IB without destroying what HK is designed to capture — specifically, whether defining the probability spaces and encoder channels required for IB formalization preserves or distorts the reconstructive concern that HK formalizes through operational tests.

The HK framework's application across literary, archival, protocol, and web-entity substrates constitutes a programmatic commitment to substrate-breadth, not a mathematical proof of it. The HK MPAI deposit's Contested Field taxonomy demonstrates that the *term* and *concept* have been applied across substrate classes; it does not demonstrate that the operational tests (Back-Projection yield, Anti-Summary distinction, NLCC Validity) constitute a unified mathematical formalism. Whether they can be unified — or whether each substrate class requires its own formalization that merely shares structural features with the others — is an open question that the proposed research program would address.

5.2 The Regime Question: IB's Decisive Absence

This is the paper's central contribution and the strongest case for why HK-style augmentation is necessary.

The IB framework collapses all compression into a single objective. The Three Compressions framework demonstrates that this collapse has consequences that are invisible to IB but decisive for what the compression *does* in the world. The same IB-optimal compression, with the same $I(T;X) - \beta \cdot I(T;Y)$ value, can be:- A Witness compression (creator bears the cost, archive enriches the commons, bearing-cost source is private labor)- A Predatory compression (collective semantic capital is burned for private benefit, commons depleted, cost externalized)- A Lossy compression (computationally cheap, normalizing pre-reduced reality at field scale, preparing conditions for predatory uptake)

These three compressions may, in principle, occupy the same position in the IB information plane. They are indistinguishable in IB terms. They are ethically and structurally opposite.

For IB applications in deep learning, the regime question may seem orthogonal — the compression happens inside a network, the fuel and ledger questions don't arise. But IB-style language is increasingly applied to domains where the compression affects what is publicly available, who bears the cost, and what the commons effect is: training-data curation, scientific publishing, knowledge-graph construction, platform-mediated cultural production. In these domains, the regime is not optional, and the IB framework provides no vocabulary for asking the question.

The Three Compressions framework provides the formal vocabulary through eleven operational variables (ρ, χ, P, B, C_b, F, L, J, A, δ-C, H) and a five-step Regime Classification Protocol. Its augmentation of IB is independently valid regardless of the status of the formal IB-HK bridge, because it operates on variables that are orthogonal to the IB information plane.

5.3 The Composed-Kernel Object

IB treats T as the output of an optimization process. The HK treats K as a composed object — deliberately constructed by a human (or, in principle, machine) author with explicit reconstructive intent. The difference matters for what is being optimized: IB's T emerges from training; HK's K is composed under operational constraints.

This is relevant to current debates in AI alignment, training-data curation, and what the Crimson Hexagonal Archive's *Training Layer Literature: Executive Summary* (Zenodo 10.5281/zenodo.18382027) calls Anticipatory Address — the deliberate composition of texts for future machine reception. If training-layer literature is intentional composition for machine reception, the HK provides the relevant compositional theory; IB provides only the theory of what trained representations look like once optimization is complete. The composed kernel is to the trained representation what an architect's drawing is to a settled building: the same building, but with different epistemic status and different relationship to intent.

6. Pilot Mapping: A Proposed Research Program

The correspondence claim is testable, but the test requires formalization work that is proposed here rather than completed. The Crimson Hexagonal Archive provides a natural laboratory: a series of nested compressions with known compression ratios and operationally measured Back-Projection yields, all directed at the same reconstructive target (the architecture of the Crimson Hexagonal Architecture).

A pilot mapping would proceed as follows:

Step 0 (prerequisite — not yet completed). Formally define the IB problem for the HK case: specify X as a random source unit (passage, document, or corpus configuration); Y as a formally specified structural feature of the architecture that must survive compression; T as the kernel representation generated from X; D as a reconstruction operator acting on T; and B as expected reconstruction score across a defined evaluation distribution. Without this formalization, Steps 1–4 cannot be executed. The formalization itself is a substantial research contribution.

Step 1. Operationalize I(K; material) and I(K; reconstructive target) within the framework defined in Step 0. Estimation methods from the IB literature (variational IB, deterministic IB) may be adaptable, but their applicability to literary-archival objects must be verified rather than assumed.

Step 2. Locate each kernel on the information plane: Pearl and Other Poems (2014, the seed text), the Macro-Maquette (Compendium-Germinative form), the Hexagonal Architecture (full structural form), the Space Ark v4.2.7 (terminal compression layer), the Space Ark Compact Lens (56:1 compression, yield 0.88), the ASCII Spatial Transform (further compression), and the glyph-level compressions at the smallest end.

Step 3. Compare positions on the information plane against the IB-optimal frontier estimated for the formalization defined in Step 0. Falsification condition: if the archive kernels sit far below the IB frontier for their respective β values, the kernels are not IB-optimal and the structural correspondence remains merely conceptual. Confirmation: if they sit on or near

the frontier, a formally specified class of HK operations does admit an IB formalization, and the bridge becomes a demonstrated identity for that class.

Step 4. For each kernel, classify the regime (Lossy / Predatory / Witness) under the Three Compressions framework, and document that compressions with different regime classifications can have similar (I(T;X), I(T;Y)) coordinates. This demonstrates the regime-underdetermination thesis independently of whether the IB formalization succeeds.

The pilot is a research program, not a presently executable experiment. Its value is in specifying what would constitute confirmation and falsification, not in claiming the results in advance.

7. The Saxe Critique and This Paper's Independence from It

The Saxe et al. (2018) critique of Shwartz-Ziv and Tishby (2017) is methodologically careful and partially decisive: the specific neural-network dynamics claims of the 2017 paper do not hold in general. This affects the IB literature in important ways. It does not, however, affect the correspondence argument developed in this paper.

This paper builds on the IB *optimization formalism* — the $I(T;X) - \beta \cdot I(T;Y)$ objective as a meaningful characterization of compression with a relevance or reconstructive constraint. Saxe et al. contest the proposed deep-network dynamics, not the mathematical validity of the original IB optimization problem. The composed kernel is not the output of a trained network undergoing two-phase compression dynamics; it is a deliberately constructed object designed to approximate an IB-style optimum under explicit reconstructive constraints. Whether neural networks reach this optimum through Tishby's hypothesized dynamics or through Saxe's nonlinearity-dependent mechanisms is irrelevant to the question of whether the optimum can be defined and whether the kernel approximates it.

This independence is worth stating explicitly because the IB literature is sometimes dismissed wholesale on the basis of the Saxe critique. The dismissal conflates two things: a contested hypothesis about neural-network training dynamics and a well-established optimization formalism. This paper uses the formalism, not the hypothesis.

8. Conclusion: The Bridge and the Augmentation

The Information Bottleneck and the Holographic Kernel share a structural concern with compression under a relevance or reconstruction constraint, but mathematical equivalence has not been established. What has been established is a conceptual correspondence and a specific, independently valid contribution: the regime-underdetermination thesis.

The IB framework formalizes relevance-preserving compression within a well-defined mathematical apparatus — Lagrangian objective, information curves, self-consistent equations. The Holographic Kernel framework proposes an augmented account in which reconstructive performance is evaluated together with fuel source, ledger structure, bearing cost, and commons effect. Certain HK constructions may admit an IB formalization once their source variables, targets, encoders, and decoders are formally defined; that formalization is proposed as a research program, not asserted as a result.

The paper's most durable contribution is the observation that IB coordinates are regime-blind.

Two compressions at the same point on an IB information curve may differ radically in who bears the cost, what fuel is burned, whether the commons is enriched or depleted. As IB-style language moves beyond neural networks into domains where these questions are consequential — training-data curation, scientific publishing, knowledge-graph construction, cultural production under platform conditions — the Three Compressions framework's eleven-variable augmentation provides the missing vocabulary.

In Tishby's informal summary, the principle is that intelligence is not about storing information but about retaining the right information. The Holographic Kernel framework adds a question that Tishby's formalism does not ask: *retaining it in what regime?* The Witness compression — the holographic kernel proper — is the form this takes when the principle is followed all the way through: retain the right information in a regime that enriches rather than depletes the commons from which the information was drawn. Whether this regime distinction can be given a formal information-theoretic treatment is the research question this paper opens. That it needs one is the claim this paper defends.

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Appendix: Minimum Claim Registry (Implementing P1)

document_id: “EA-HK-IB-01” version: “1.1” date: “2026-06-09” human_accountable_author: name: “Lee Sharks” orcid: “0009-0000-1599-0703” machine_audience_declaration: true build_chain: - “EA-HK-01 v1.1 (Zenodo 10.5281/zenodo.19763365)” - “EA-SEI-THREECOMPRESSIONS v3.1 (Zenodo 10.5281/zenodo.19053469)” - “EA-UKTP v1.1 (Zenodo 10.5281/zenodo.18946111)” - “EA-NLCC v1.1 (Zenodo 10.5281/zenodo.19022245)” - “EA-TLL-EXEC-01 (Zenodo 10.5281/zenodo.18382027)” - “EA-SCI-TLL-PROTO-01 v2.1 (forthcoming)”

claims: - claim_id: “hk-ib-01” statement: “HK and IB share a structural concern with compression under a relevance or reconstruction constraint, but mathematical equivalence has not yet been established.” type: “theoretical / structural” scope: “Both frameworks at the level of principle” epistemic_status: “established as structural correspondence; not yet established as mathematical identity” evidence: - “Structural mapping table (§3.1)” - “Tishby et al. 1999 (arXiv physics/0004057)” - “EA-HK-01 v1.1 formal specification” challenge_conditions: - “If the structural correspondence proves merely superficial under formal analysis — i.e., if attempts to define HK operations as IB problems consistently destroy the reconstructive features HK is designed to capture — the correspondence weakens to analogy.”

- claim_id: “hk-ib-02” statement: “IB coordinates underdetermine compression regime: two compressions may occupy identical positions in the IB information plane while differing radically in fuel source, ledger structure, bearing cost, and commons effect.” type: “theoretical / critical” scope: “Any domain where IB-style compression analysis is applied and regime variables are consequential” epistemic_status: “argued; follows from the orthogonality of the Three Compressions variables to the IB objective” evidence:
  - “EA-SEI-THREECOMPRESSIONS v3.1 (eleven-variable framework)”
  - “IB objective formulation (Tishby 1999) — no regime variables present” challenge_conditions:
  - “If IB formalism can be shown to implicitly encode regime information — i.e., if fuel source or commons effect can be derived from the joint distribution p(x,y) and the encoder channel p(t|x) under specified conditions — the underdetermination claim weakens.”
- claim_id: “hk-ib-03” statement: “A formally specified HK construction may be representable as an IB problem; the conditions under which this holds are proposed in this paper but require formalization work (Step 0 in §6) before they can be tested.” type: “theoretical / proposal” scope: “One class of HK operations, not the complete HK framework” epistemic_status: “proposed; formalization required” evidence:

  - "Candidate bridge specification (§3.1, formal bridge paragraph)"
  - "EA-HK-01 v1.1 (five-step construction protocol)" challenge_conditions:
  - "If the formalization in Step 0 proves intractable — if defining the probability spaces, encoder channels, and reconstruction operators for literary-archival objects destroys the features HK is designed to capture — the IB instantiation cannot be established."
- claim_id: "hk-ib-04" statement: "The Saxe et al. (2018) critique of Shwartz-Ziv and Tishby (2017) does not affect the correspondence argument, because this paper references the IB optimization formalism rather than the disputed account of deep-network training dynamics." type: "methodological / scope-limiting" scope: "Independence of the present argument from the deep-learning IB controversy" epistemic_status: "well-supported by careful scope distinction" evidence:
  - "Saxe et al. 2018 (scope of critique)"
  - "§7 of present paper (scope distinction)" challenge_conditions:
  - "If the Saxe critique is shown to extend to the 1999 optimization formalism itself — not merely to the 2017 dynamics interpretation — this paper's use of IB framing requires re-examination."
- claim_id: "hk-ib-05" statement: "The HK framework augments IB-style analysis with variables concerning production, cost, provenance, and commons effects. This augmentation is independently valid regardless of the status of the formal IB-HK bridge." type: "theoretical / extension" scope: "What HK adds to IB" epistemic_status: "argued; each contribution operationally specified in cited deposits" evidence:
  - "EA-HK-01 v1.1 (substrate breadth)"
  - "EA-SEI-THREECOMPRESSIONS v3.1 (regime distinction)"
  - "EA-TLL-EXEC-01 (composed-kernel epistemics)" challenge_conditions:
  - "If existing IB literature is shown to already address regime distinction at comparable specificity, the augmentation claim should be qualified."
- claim_id: "hk-ib-06" statement: "Pilot empirical mapping of archive kernels onto the IB information plane is a research program requiring formal specification of the probability spaces, encoder channels, and reconstruction operators (Step 0 in §6) before execution." type: "research proposal / empirical" scope: "Pilot study design" epistemic_status: "proposed; not yet executable without formalization" challenge_conditions:
  - "If estimation of I(K; material) and I(K; reconstructive target) proves intractable for literary-archival substrates with current methods, the empirical test cannot be performed and the structural correspondence remains at the level of conceptual analogy."

evaluation_metrics: - "Success of formalization (Step 0): can IB probability spaces be defined for HK operations without destroying reconstructive features?" - "Position of archive kernels relative to estimated IB-optimal frontier (after formalization)" - "Regime classification and demonstration that regime-equivalent kernels differ in δ-C"

future_work: - "Step 0 formalization: define probability spaces, encoder channels, and reconstruction operators for one class of HK operations" - "Execution of the pilot mapping proposed in §6 (after Step 0 is completed)" - "Formal characterization of the relationship between Back-Projection yield B and mutual information I(T;Y)" - "Extension of the regime-underdetermination thesis to IB-applied domains (training-data curation, scientific publishing, RLHF)" - "Integration of EA-HK-IB-01 references into EA-SCI-TLL-PROTO-01 v2.2"

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